APPLICATION OF A BASIC ALGORITHM TO SOLVE PROCESS CONTROL SYSTEMS

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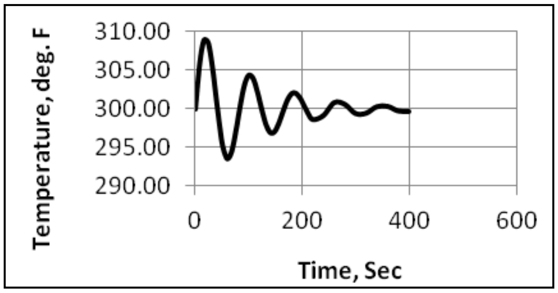

Quite often, an engineer is confronted with the task of analyzing dynamic or transient type processes. These processes are generally modeled by using ordinary differential equations or partial differential equations. These differential equations can be highly nonlinear in nature, and analytical solutions are either very difficult or even impossible to achieve. As a result, numerical techniques must often be employed to integrate these systems.

In our previous paper (1) in this series of two papers, we employed a BASIC algorithm to perform the necessary integrations for several dynamic or transient-type processes. These illustrations considered only open-loop type problems. For instance, if the concentration of a reactant species in the feed stream to a CSTR reactor was perturbed, we proceeded to model and observe the time-varying response of the remaining reactant concentration in the effluent stream.

In this second paper of the series, we extend the application of the BASIC algorithmic scheme to closed-loop process control systems employing negative feedback. The first page of the paper describes a typical process control scenario for a simple liquid level-flow system. A brief review of the various general type ideal controller schemes is presented first followed by a review of the basis for the algorithmic structure. Then we present three specific illustrations which simulate process control for the following dynamic processes:

1. A heated tank with inflow and outflow of liquid where it is desired to control the temperature of the outflow stream.

2. Two interacting tanks involving a liquid level-flow network where we wish to control the liquid level in the second tank.

3. Two CSTR reactors operating in series where it is required that we control the concentration of the remaining reactant in the effluent from the second reactor.

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APPLICATION OF A BASIC ALGORITHM TO SOLVE PROCESS CONTROL SYSTEMS

I. Introduction

Previous Paper In our previous study (1), we employed a BASIC algorithm to solve several dynamic or transient-type processes. These illustrations considered only open-loop type problems. For instance, if the concentration of a reactant species in the feed stream to a CSTR reactor was perturbed, we modeled and observed the immediate response in the remaining reactant concentration in the effluent stream.

This Study In this second follow up paper, we extend our application of the BASIC algorithmic scheme to closed-loop process control systems employing negative feedback.

As a typical example, consider the simple liquid level-flow system shown below (see Diagram 1):

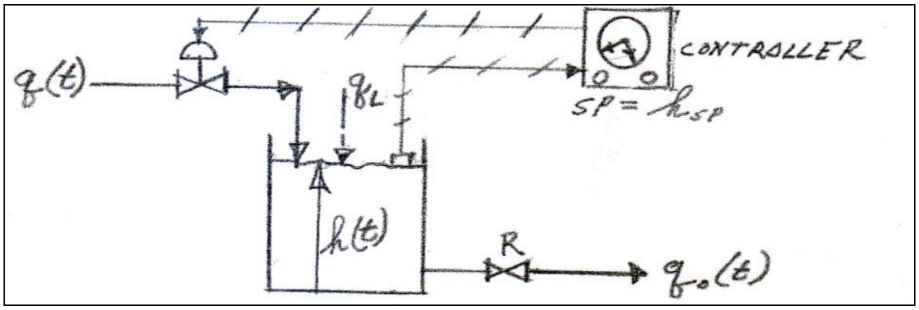

Diagram 1 Simple liquid level-flow system

The system consists of a tank of uniform cross-sectional area A being fed by a liquid stream with flow rate q(t). Attached to the bottom of the tank is an exit line with flow rate $q_o(t)$ and resistance R such as a valve, weir, orifice etc. Initially the inlet and discharge flow rates are equal, and a certain liquid level h(t) in the tank is maintained. Now the system is perturbed by adding a second feed steam q_L. When this happens, the liquid level is obviously also perturbed and begins to rise. In this case, it is desirable to maintain or control the original liquid level in the tank (h_{SP}) by means of a controller. Here h_{SP} is referred to as the controller set point. If the indicated liquid level h(t) differs from the desired level, the controller senses the difference or error, $\epsilon = h_{sp} - h$, and changes the original feed flow rate q(t) so as to reduce the magnitude of ϵ.

A very brief review and discussion of the various ideal controller schemes is given next.

II. Review of Different Controller Types

Proportional Control An ideal proportional controller produces and output signal which is simply proportional to the error ϵ. When the controller is acting upon a valve, its output signal usually consists of a pressure varying from 3 psi (valve fully closed) to 15 psi (valve fully open). In this case proportional action is expressed by,

$$p = p_s + K_c \varepsilon \qquad (1)$$

where p = output pressure signal, psi

K_c = gain or sensitivity

ϵ = error = set point - measured variable

p_s = constant or steady state pressure signal

Here the error ϵ, the difference between the set point and the signal from the measuring element, can be expressed in any suitable set of units.

Proportional-Integral (PI) Control PI control is described by the relationship,

$$p = p_s + K_c \varepsilon + \frac{K_c}{\tau_I} \int_0^t \varepsilon \, dt \qquad (2)$$

where K_c = the gain

τ_I = integral time e.g. minutes

p_s = steady state pressure

With PI control we have added to the proportional action a second term which is proportional to the time integral of the error. The values of K_c and τ_I are varied or adjusted by two separate knobs in the controller itself.

Certain controller manufacturers prefer to use the term reset rate, which is defined as the reciprocal of τ_I.

Proportional-Derivative (PD) Control The action of a PD controller is represented by Equation 3,

$$p = p_s + K_c\,\varepsilon + K_c\,\tau_D \frac{d\,\varepsilon}{d\,t} \qquad (3)$$

where K_c = gain

τ_D = derivative time e.g. minutes

p_s = steady state pressure signal

With PD control is added to proportional control another term which is proportional to the time derivative of the error. Once again, the values of K_c and τ_D may be varied separately by knobs in the controller. Alternative terms which describe this derivative action are rate control or anticipatory control.

PID Control This mode of control is simply a combination of all three modes of control and is given by the expression,

$$p = p_s + K_c\,\varepsilon + K_c\,\tau_D \frac{d\,\varepsilon}{d\,t} + \frac{K_c}{\tau_I}\int_0^t \varepsilon\,dt \qquad (4)$$

Here the controller has three knobs for adjusting K_c, τ_D and τ_I.

Nature of Various Controller Actions There are definite practical motivations for the use of the ideal controller modes described above. The curves shown below illustrate the behavior for a typical feedback control system employing different kinds of control when it is subjected to a permanent disturbance of some kind (see Figure 1).

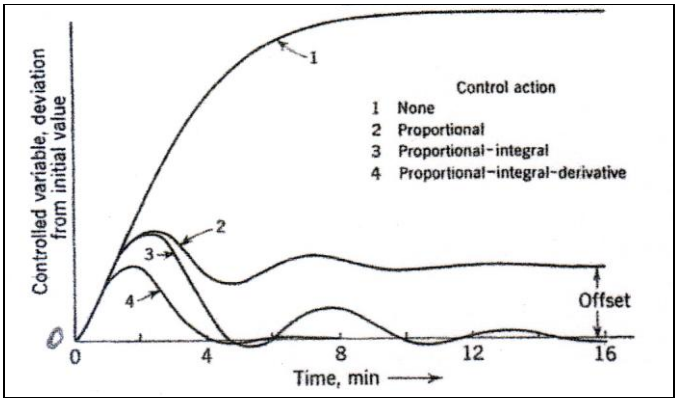

Figure 1 Behavior for typical feedback control systems

For the open loop case (K_c = 0) the values of the controlled variable rises at time t ≥ 0 owing to the disturbance. It eventually reaches a new steady state value. With control action, the control system begins to take action in order to maintain the controlled variable close to the value (set point) which previously existed before the disturbance was initiated.

When only proportional action is used, the control system immediately begins to arrest the rise of the controlled variable and eventually bring it to rest at a new steady state value. The difference between the new steady state value and the original value is called offset.

The addition of integral action (see the PI curve) eliminates the offset, and the controlled variable ultimately returns to its original value. However, the PI action results in somewhat more oscillatory behavior.

The addition of derivative control to the PI action (PID) provides a distinct improvement in the response. In this case, the rise in the controlled variable is arrested more quickly and is returned to the original value very rapidly with minimal oscillations.

A more comprehensive discussion and treatment of process control theory can be found in such outstanding textbooks as Coughanowr and Koppel (2), Harriott (3) and Shinskey (4). A discussion on control system stability is beyond the scope of this writing. A complete discussion on stability analysis techniques, such as frequency response (Bode plots) and root-locus tests, can readily be found in the above reference books.

III. Review of Basis for the Algorithmic Structure

Runge-Kutta Formulae In our first paper (1) we described several numerical methods for solving first order ordinary differential equations subject to a specified set of initial conditions. Of these methods the fourth order Runge-Kutta formulas represented the most accurate and efficient procedure for performing the required numerical integrations. In this paper we employ fourth order RK only for solving the ODE's which model the dynamic processes involved. These ODE's along with he appropriate expression for corrective action such as given by Equations 1-4 were used to simulate process control for three dynamic processes involving:

1. A heated tank with inflow and outflow of liquid where it was desired to control the temperature of the outflow stream.

2. Two interacting tanks involving a liquid level-flow system where we wish to control the liquid level in the second tank.

3. Two CSTR reactors operating in series where it is required that we control the concentration of the remaining reactant in the effluent from the second reactor.

A review of the fourth order Runge-Kutta process is described below. For a general first-order ODE of the form dy/dx = f(x,y) subject to the initial condition that when $x = x_o$, $y = y_o$, the fourth-order RK process is represented by the following equation set:

$$k_1 = h\, f\left(x_n, y_n\right) \qquad (5)$$

$$k_2 = h\, f\left(x_n + h/2, y_n + k_1/2\right) \quad (6)$$

$$k_3 = h\, f\left(x_n + h/2,\ y_n + k_2/2\right) \qquad (7)$$

$$k_4 = h\, f\left(x_n + h,\ y_n + k_3\right) \qquad (8)$$

$$y_{n+1} = y_n + \left(k_1 + 2k_2 + 2k_3 + k_4\right)/6 \quad (9)$$

$$at\ \ x_{n+1} = x_n + h$$

$$for\ \ n = 1, 2, 3 \ldots ; O\left(h^5\right)$$

The use of these equations is quite simple and straightforward. Their use is quite popular with program analysts. Starting with $y = y_o$ when $x= x_o$ and a selected interval h in x, k_1 must be calculated before k_2 can be evaluated; k_2 before k_3 and so on. The notation $f(x_n + h/2, y_n + k_1/2)$ means that f(x,y) is being evaluated specifically at,

$$x = x_n + h/2 \ ; \quad y = y_n + k_1/2$$

The resulting values of $k_1 \ldots k_4$ are then substituted into the final equations (Eqn. 9) to yield y_1 at x_1; y_2 at x_2; ... y_{n+1} at x_{n+1}. The truncation error associated with this process is very small and is of the order of the fifth power of the selected interval h in x.

The Runge-Kutta process can readily be applied to the numerical integration of simultaneous sets of first-order ODE's subject to a corresponding set of initial conditions for each dependent variable. The simplest situation consists of two ODE's involving two dependent variables, say y and p, and a single independent variable x, usually representing the lapse of time.

$$\frac{dy}{dx} = y' = f_1\left(x, y, p\right) \qquad (10)$$

$$\frac{dp}{dx} = p' = f_2\left(x, y, p\right) \qquad (11)$$

$$with\ i.c. \qquad y(x_o) = y_o \ ; \quad p(x_o) = p_o$$

The corresponding fourth-order RK expressions for this ODE set are as follows:

$$k_1 = h\, f_1(x_n, y_n, p_n) \quad (12)$$

$$l_1 = h\, f_2(x_n, y_n, p_n) \quad (13)$$

$$k_2 = h\, f_1(x_n + h/2, y_n + k_1/2, p_n + l_1/2) \quad (14)$$

$$l_2 = h\, f_2(x_n + h/2, y_n + k_1/2, p_n + l_1/2) \quad (15)$$

$$k_3 = h\, f_1(x_n + h/2, y_n + k_2/2, p_n + l_2/2) \quad (16)$$

$$l_3 = h\, f_2(x_n + h/2, y_n + k_2/2, p_n + l_2/2) \quad (17)$$

$$k_4 = h\, f_1(x_n + h, y_n + k_3, p_n + l_3) \quad (18)$$

$$l_4 = h\, f_2(x_n + h, y_n + k_3, p_n + l_3) \quad (19)$$

Then

$$y_{n+1} = y_n + (k_1 + 2k_2 + 2k_3 + k_4)/6 \quad (20)$$

$$p_{n+1} = p_n + (l_1 + 2l_2 + 2l_3 + l_4)/6 \quad (21)$$

Obviously, this procedure can be generalized and applied to any number of simultaneous first-order ODE's by introducing more parameters i.e. k, l, m, n...etc. as required.

The BASIC DEF FN- Function Many times a rather complex function may need to be called upon many times in a BASIC program. In these cases it is very advantageous to use the DEF statement followed by the name of the defined function, which must consist of three letters, the first two of which are FN. QBASIC routines generally allow you to define up to 26 functions e.g. FNA, FNB, FNC, etc.

As an example, the great utility of such a function can be seen in a program where we may require repetitive evaluation of the function,

$$e^{-x^2}$$

In this case we can represent this function by the DEF statement,

```
30 DEF FNA(X) = EXP (- X ^ 2)
```

At some other point in the program we may call for various values of the function by FNA(0.1), FNA(3.45), FNA(A + 2), etc.

The DEF statement may occur anywhere in the program, and the expression to the right of the equal sign may be any formula which can be fitted onto one line. It can involve other variables besides the one denoting the argument of the function. Thus, assuming FNR is defined by,

$$70\ DEF\ FNR(X) = SQR\left(2 + LOG(X) - EXP(Y*Z)\right)*(X + SIN(2*Z))$$

if we have previously assigned values for Y and Z, we can ask for FNR(2.175) where X = 2.175. New values of Y and Z can be assigned or calculated before the next use of FNR. Often such complicated functions must be calculated at several different points within the program such as in a subroutine. For these functions, the GOSUB statement may be quite useful.

For the study here, we will see that such function calls are extremely useful for conducting the repetitive calculations required by the Runge-Kutta formulas. Suppose we wish to solve the first-order nonlinear ODE,

$$\frac{dy}{dx} = 2.3746 - 1.8y - 0.5746\,y^3 \quad with\ i.c.\ y(0) = 0$$

This ODE can be defined once in the program by the DEF function statement.

$$DEF\ \ FNA(Y) = 2.3746 - 1.8*Y\ - 0.5746*Y\,^\wedge 3$$

This functional form is then conveniently embedded into the RK formulas, Eqns. 5-8, for repetitive evaluation. This application will be clearly demonstrated in the Illustrations to follow.

IV. Illustrations

Illustration 1 In the first application we have a liquid stream at temperature T_i flowing at a rate of W lbs/sec. It is desired to heat this stream to a specified higher temperature T_R. This task will be accomplished by inputting this colder stream into a well-agitated tank which is equipped with a heating device. A simple sketch of the open loop heating system is shown below:

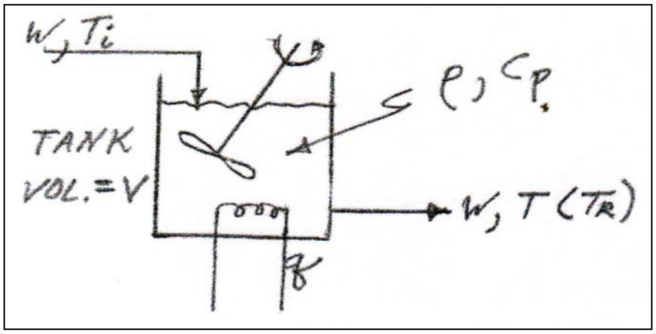

Diagram 2 Open loop heating system

It will be assumed that the agitation is sufficient to ensure that the entire fluid in the tank will be at a uniform temperature T. The heated fluid is removed from the bottom of the tank at flow rate W and is obviously the product or effluent of this heating process. In summary, the following operating conditions are assumed to prevail:

1. The mass of fluid (holdup) retained in the tank remains constant.

2. The desired temperature of the effluent is the same as that of the fluid in the tank.

3. Physical properties of the fluid such as heat capacity and density are constant in spite of any traverses in the system temperature.

When the process is operating at steady state, the energy balance around the tank is written as,

$$q_s = W\,C_p\left(T_R - T_{is}\right) \qquad (22)$$

q_s is the heat input needed to maintain this steady state condition. T_{is} is the normal inlet temperature to the tank, and T_R is the desired steady state effluent temperature.

It should be quite apparent that, if the heater is set to deliver only a constant output q_s, a change in the process conditions will cause the tank temperature to depart from a value of T_R. Typically, the process condition that will change is the inlet temperature T_i. As a result, the heater must be designed so that its energy output may be varied as required to maintain T at or as near as possible to T_R,

In order to be able to make an intelligent estimate of how the controller decisions must be made in advance, it is necessary to develop a mathematical model of how the temperature T varies in response to changes in T_i and q. The model will consist of an unsteady state or transient energy balance for the process.

$$Heat\ In - Heat\ Out = Heat\ Accum. \quad (23)$$

$$Heat\ In = W\,C_p\left(T_i - T_{ref}\right) + q \quad (24)$$

$$Heat\ Out = W\,C_p\,(T - T_{ref}) \quad (25)$$

$$Heat\ Accum. = \rho V\,C_p\,\frac{dT}{dt} \quad (26)$$

where each quantity above has net units of energy/time and,

ρ = fluid density, lbs/cuft

V = volume of fluid in the tank, cuft

W = inlet or outlet stream flow rate, lbs/sec

C_p = liquid heat capacity, Btu/lb-deg. F

t = elapsed time, sec

When Equations 24-26 are inserted into Equation 23, we get,

$$W\,C_p\left(T_i - T\right) + q = \rho V\,C_p\,\frac{dT}{dt}$$

$$or \qquad \frac{dT}{dt} = C_1\left(T_i - T\right) + \frac{q}{C_2} \quad (27)$$

$$where \qquad C_1 = \frac{W}{\rho V}\ ;\ C_2 = \rho V\,C_p$$

The required controller for this service must use the existing values of T and T_R (set point) to adjust the heat input according to a predetermined relationship. This relation consists of the difference between these temperatures i.e. T_R - T which is called the error or ε. The sole purpose of the controller is to steer the operating system in such a direction that this error becomes zero.

In this study we will only look at proportional-integral (PI) control. For the present Illustration 1, the controller relation for adjusting the heat input will be,

$$q = q_s + K_c \varepsilon + \frac{K_c}{\tau_I} \int_0^t \varepsilon \, dt \qquad (28)$$

where $q_s = W\, C_p\, (T_R - T_{is})$

$\varepsilon = T_R - T$ with T_R = set pt. discharge temperature

K_c = proportional gain

τ_I = integral time e.g. sec.

The entire process controlled heater/tank system is depicted below in Diagram 3:

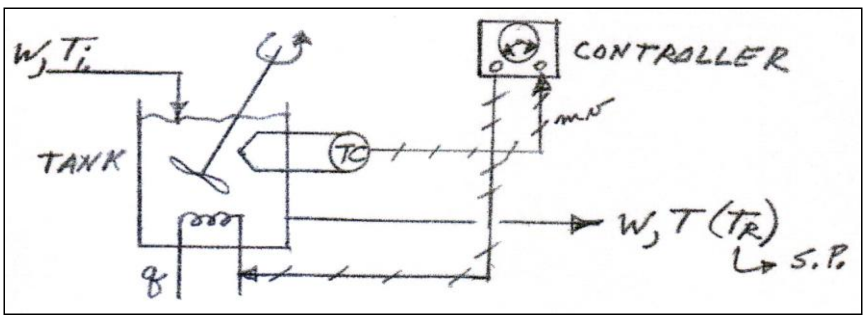

Diagram 3 Entire process controlled heater/tank system

The feedback signal from the temperature measuring device, e.g. a thermocouple, is an EMF in units of millivolts (mv). So, realistically, the controller gain K_c should have units of Btu/sec-mv.

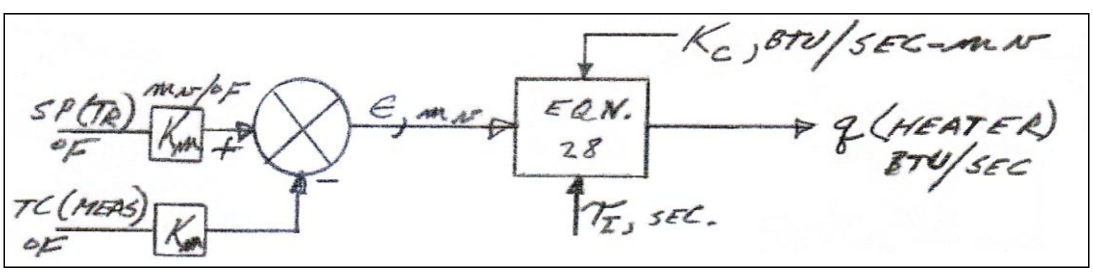

Diagram 4 Controller

The measuring device here (TC) basically changes the units between input and output signals and is referred to as a transducer. The gain K_m associated with this measurement has net units of mv/deg. F. For our purposes here, we will simply lump this gain in with the controller gain and input trial values of K_c in overall units of Btu/sec-deg. F.

The specific numerical parameters for our problem here are:

Physical properties:

$W = 100$ lbs/sec — $C_p = 1.0$ Btu/lb- deg. F

$V = 100$ cu ft — $\rho = 60$ lbs/cu ft

$T_{is} = 100$ deg. F

$T_R(SP) = 300$ deg. F

We will be looking at two specific cases for controlling the tank discharge fluid temperature.

Case a: With the system operating initially at steady state, let us study the control scenario for maintaining the specified SP discharge temperature of 300 deg. F when the inlet stream temperature is suddenly increased from 100 deg. F to 150 deg. F.

Case b: Perform a control scenario analysis for the system if we keep the inlet temperature fixed at 100 deg. F but wish to achieve a new discharge stream set point temperature of 350 deg. F.

The BASIC program listing (CNTR1A.BAS) for Case a is given in Table 1. Line 20 in tandem with DATA Line 500 reads in the initial operating conditions and physical properties. From Lines 30 and 510 we read in the integration interval, total time duration, and the printout frequency parameter. For the specific problem to be executed here, we have chosen a total time span of 400 sec. So therefore,

$$H = 0.1 \text{ sec} \quad ; \quad M = 400 \text{ sec} \quad ; \quad S = 200$$

From the formula listed in Line 230 we have,

$$\frac{400}{(0.1)(200)} = 20$$

As a result, Line 370 will print the results line-by-line every (0.1)(20) or 2 seconds up to a total time span of 400 seconds.

Line 40 computes the initial steady state heater input via Equation 22 for the condition where the inlet stream temperature is 100 deg. F.

$$q_s = (100)(1.0)(300 - 100) = 20,000\ Btu/Sec$$

This value is printed out via Line 50.

Lines 210 and 229 provide the DEF function statements for the two ODE's to be solved here. The first one defines Equation 27, and the second one is needed to evaluate the time integral of the error $\varepsilon = T_R - T$ present in Equation 28. Thus, we have for the first ODE:

210 DEF FNA(T) = C1*(TI - T) + Q/C2

The second ODE represents the following derivative,

$$\frac{dZ}{dt} = \varepsilon = T_R - T \quad and\ therefore\ Z = \int_0^t \varepsilon\, dt$$
$$after\ integration\ via\ Line\ 330\ of\ the\ program.$$

So Line 229 of the program is,

229 DEF FNB(T) = TR - T

Lines 240 through 330 perform the repetitive fourth-order Runge-Kutta integration procedure to generate the required values for T and Z. Line 340 (Eqn. 28 of the text) then generates a new or corrected value for the heater input which is next fed back to Line 210 so that a new value of T can be calculated for the next time interval.

We tried various values for the proportional gain K_c and the integral time τ_I and arrived at a satisfactory control scenario for the specific values:

K_c = 7.0 Btu/sec- deg. F

τ_I = 0.2 seconds

Table 2 (CNTR1A.OUT) lists the output covering the first 100 sec for the specific run employing the above controller settings. Figure 2 shows the temp./time profile for the tank fluid discharge temperature, and Figure 3 displays the variation of the heater input to the tank fluid over the same lapse of time. Both profiles exhibit oscillatory behavior as is expected when the PI mode of control is used. After about 200 seconds the system begins to stabilize nicely, and the set point temperature of 300 deg. F is basically reached. The heater input lines out at a value close to 15,000 Btu/sec which is consistent with the new steady state conditions reached when the inlet stream temperature is increased from 100 deg. F to 150 deg. F.

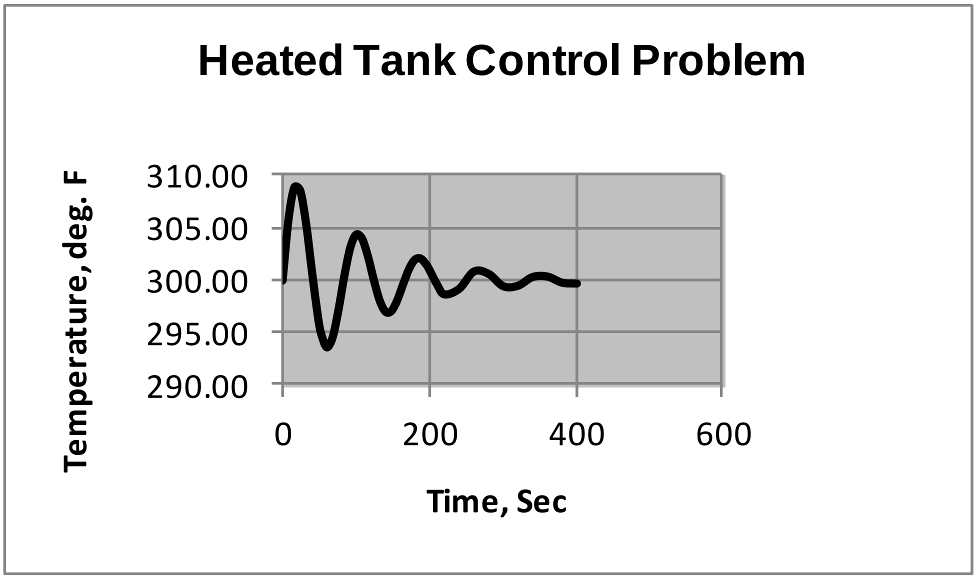

Figure 2 Illus. no. 1 - Case A (PI control action)

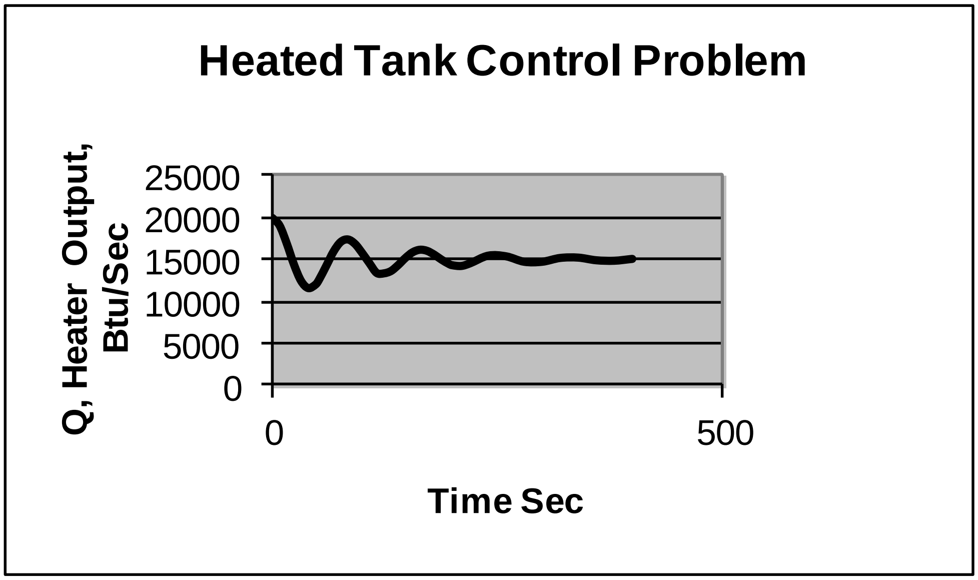

Figure 3 Illus. no. 1 - Case A (PI control action)

Studies were also conducted with Case a for cases where no integral action was present. We reran the same problem for K_c = 7.0 and 100 Btu/sec-deg. F and also for K_c = 0, the open loop case. Figure 4 gives the T vs. t profile and Figure 5 the q vs. t profile for the case where K_c = 100 with no integral action. Figure 6 shows the T vs. t plot for the open loop case (K_c = 0, no integral action). In every case here a steady state error (offset) is obtained.

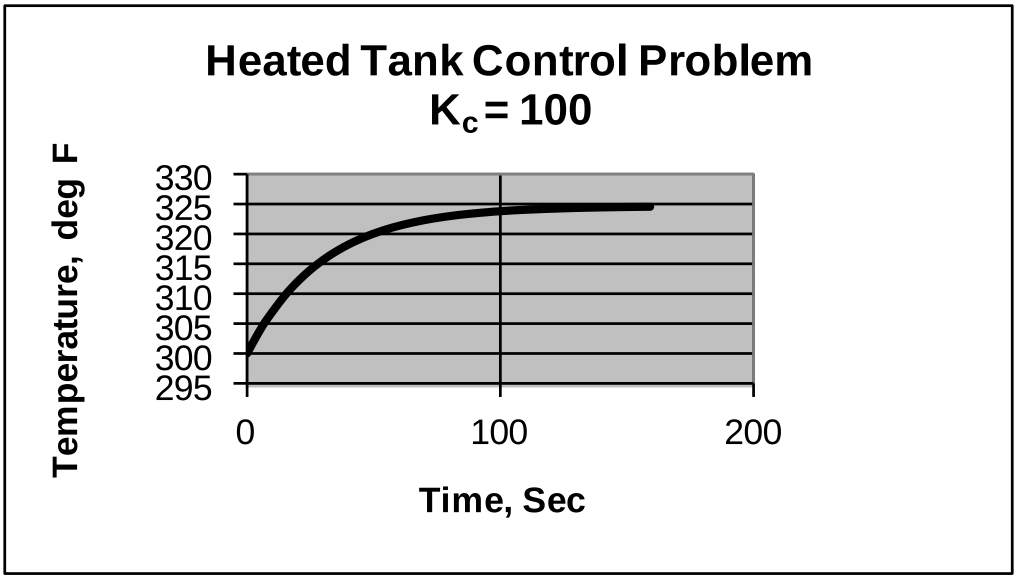

Figure 4 Illus. no. 1 - Case A (no integral action)

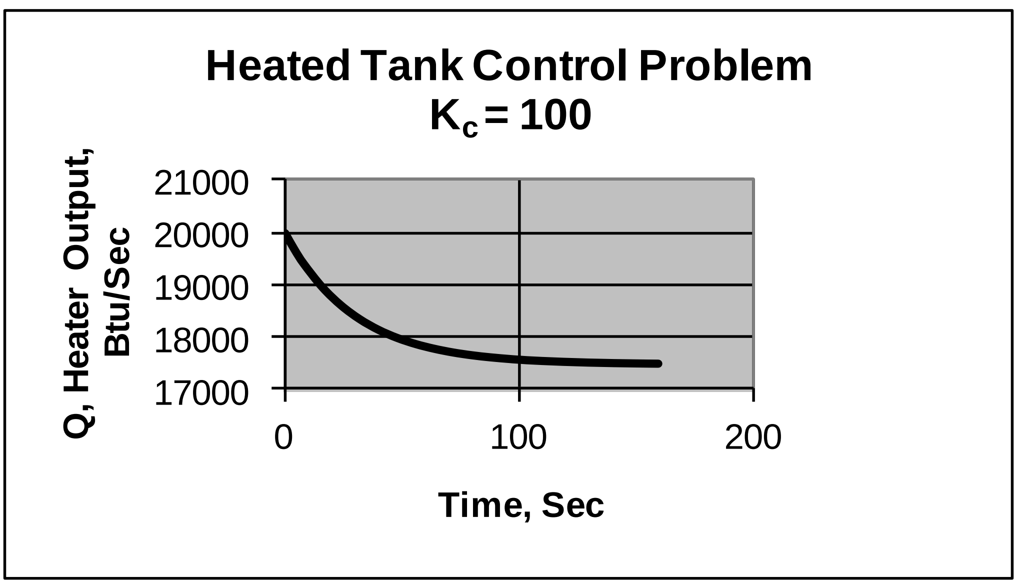

Figure 5 Illus. no. 1 - Case A (no integral action)

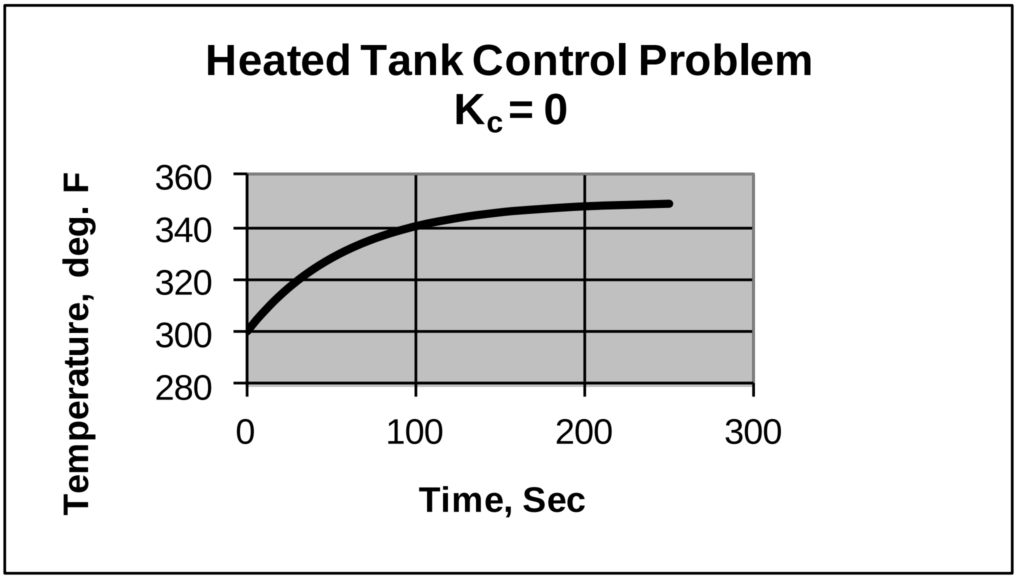

Figure 6 Illus. no. 1 - Case A (no integral action)

In summary:

K_c	Set Pt. deg. F	Steady state Temp. Deg. F	Offset, deg. F
0	300	350	50
7.0	300	347	47
100.0	300	325	25

With Case b we will maintain the inlet stream temperature at 100 deg. F but now specify a new and higher discharge set point temperature of 350 deg. F. All other properties such as system flow rate, tank volume, and the fluid physical properties are kept the same as for Case a.

Tables 3 and 4 list the BASIC program (CNTR1B.BAS) and corresponding output (CNTR1B.OUT) for Case b using the same controller settings as used in Case a, namely $K_c = 7.0$ and $\tau_I = 0.2$ seconds. Figures 7 and 8 provide the T vs. t and q vs. t profiles respectively for this run (Case B). Once again, oscillatory behavior with eventual zero offset is achieved. The system stabilizes out nicely after about 200 seconds have elapsed. The final steady state heater input becomes,

$$q_s \text{ (new)} = (100)(1.0)(350 - 100) = 25{,}000 \text{ Btu/sec}$$

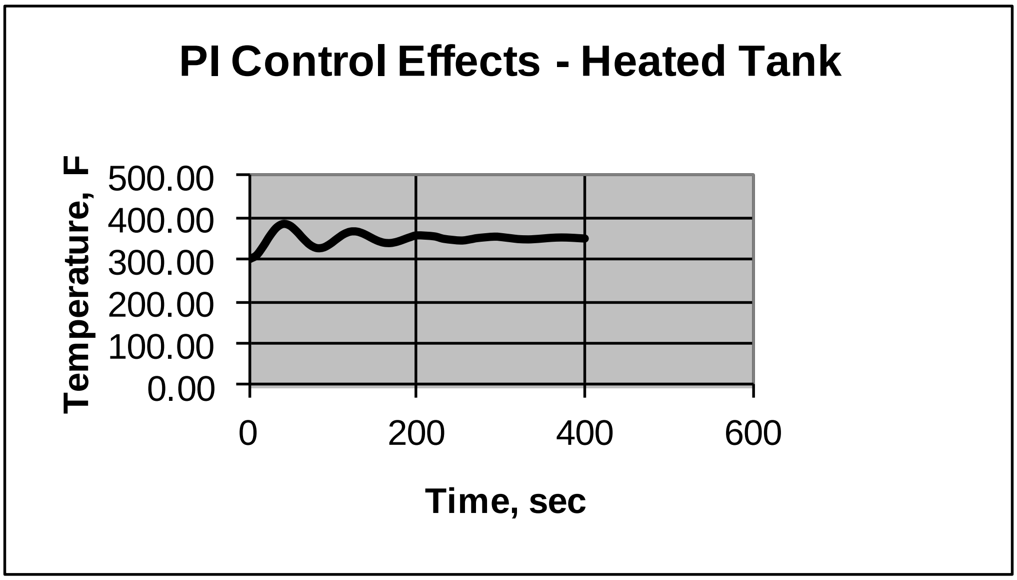

Figure 7 Illus no. 1 - Case B (PI control action)

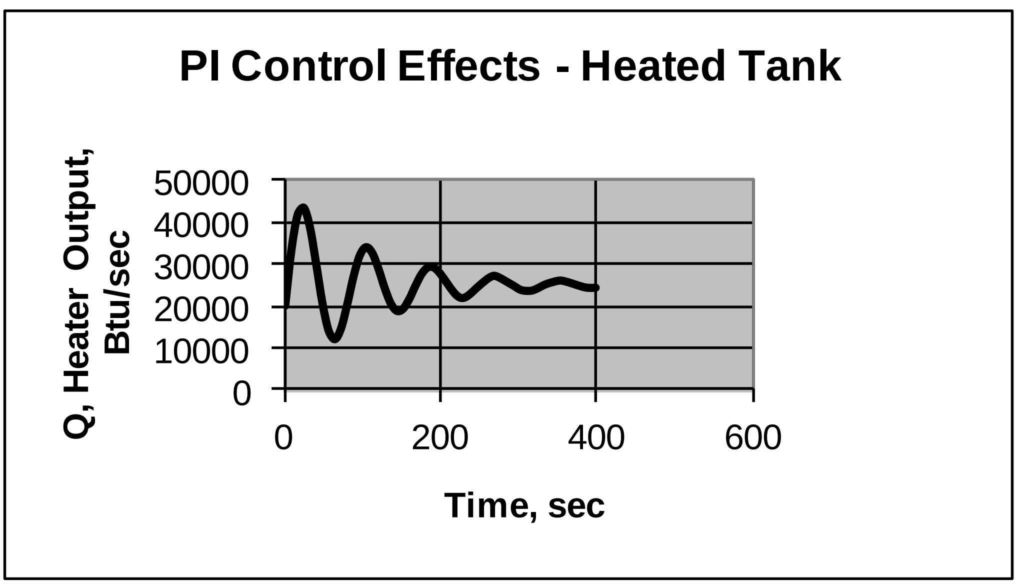

Figure 8 Illus no. 1 - Case B (PI control action)

Illustration 2 The next example involves a two-tank interacting liquid level flow network as depicted below:

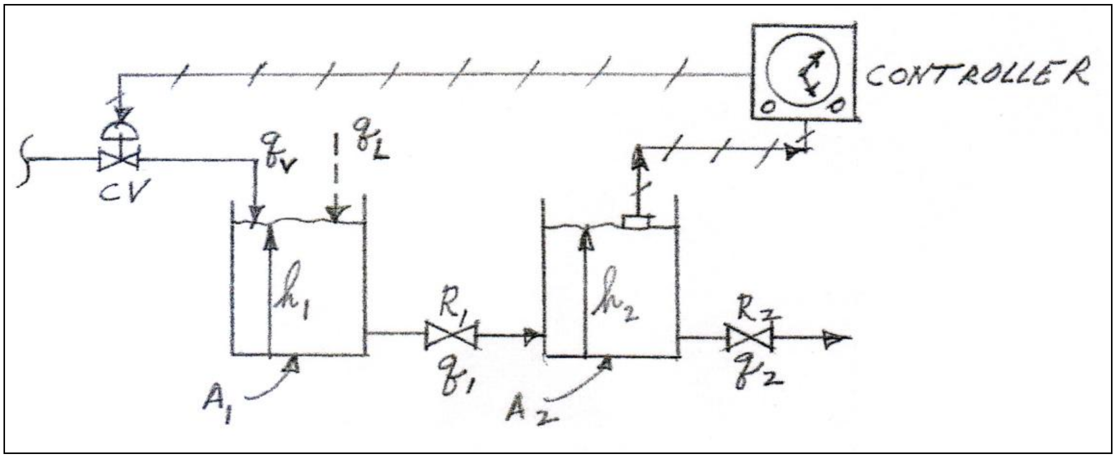

Diagram 5 Two-tank interacting liquid level flow network

First, we define all of the pertinent parameters shown above before developing the appropriate process control model.

q_v = volumetric flow rate to tank no. 1, CFM (main feed flow)

A_1 = uniform cross-sectional area of tank no. 1, sq ft

h_1 = liquid level in tank no. 1, feet

R_1 = linear flow resistance in line connecting tanks no. 1 and 2, ft/CFM

q_1 = volumetric flow rate in the interconnecting line, CFM

A_2 = uniform cross-sectional area of tank no. 2, sq ft

h_2 = liquid level in tank no. 2, feet

R_2 = linear flow resistance in the exit line from tank no. 2, ft/CFM

q_2 = volumetric flow rate exiting the second tank, CFM

Initially the flow network is operating at steady state such that,

$$q_v = q_1 = q_2$$

and both tank liquid levels are unchanging. Now suddenly, a second inlet flow stream at a rate of q_L CFM is introduced to the first tank. Clearly, the system will be perturbed and will seek a new steady state condition whereby h_1 and h_2 will assume entirely new values. However, it is our desire to keep the liquid level h_2 in tank no. 2 the same as it was before the flow disturbance was initiated. We propose to achieve this goal by utilizing a PI controller which senses the instantaneous liquid level in tank 2 and then provides corrective action by changing the main feed flow rate q_v to tank no. 1 via a control valve.

In this case, the controller set point would be h_{2sp}. If the measured level $h_2(t)$ differs from the set point, the controller senses the error $\varepsilon = h_{2sp} - h_2$ and changes $q_v(t)$ accordingly.

The first step to perform in developing the controller scheme is to write the unsteady state volumetric flow balances around each tank. The flow-head relationships for the two linear resistances are given by the expressions:

$$q_1 = \frac{h_1 - h_2}{R_1} \qquad (29)$$

$$q_2 = \frac{h_2}{R_2} \qquad (30)$$

where the flow rate between interacting tanks is dependent upon the difference between h_1 and h_2. Now, assuming a constant density liquid flowing system, the unsteady state volumetric flow balances around tanks no. 1 and 2 lead to two first-order ODE's:

$$q_v + q_L - q_1 = A_1 \frac{d\,h_1}{d\,t}$$

$$q_v + q_L - \frac{h_1 - h_2}{R_1} = A_1 \frac{d\,h_1}{d\,t}$$

or

$$\frac{d\,h_1}{d\,t} = \frac{q_v + q_L}{A_1} - \frac{h_1 - h_2}{R_1\,A_1} \qquad (31)$$

$$q_1 - q_2 = A \frac{d\,h_2}{d\,t}$$

$$\frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2} = A_2 \frac{d\,h_2}{d\,t}$$

or

$$\frac{d\,h_2}{d\,t} = \frac{h_1 - h_2}{R_1\,A_2} - \frac{h_2}{R_2\,A_2} \qquad (32)$$

Here q_L represents the flow disturbance introduced as an additional feed to tank no. 1.

Once again, we will apply PI control to the system. The controller action for adjusting the major feed flow rate is represented by the relation,

$$q_v = q_{vs} + K_c\,\varepsilon + \frac{K_c}{\tau_I}\int_0^t \varepsilon\,dt \qquad (33)$$

where $\varepsilon = h_{2SP} - h_2$

$K_c = proportional\ \ gain,\ CFM / ft$

$\tau_I = \text{int}\,egral\ \ time, \text{min}$

The control elements are also shown on the sketch above. The numerical parameters to be assigned to this second illustration are as follows:

A_1 = 1 sq ft	R_1 = 2 ft/CFM
A_2 = 1 sq ft	R_2 = 1 ft/CFM
h_{1s} = 9 ft	h_{2s} = 3 ft set pt.

The liquid levels above represent the initial steady state levels in tanks no. 1 and 2 respectively. To comply with these initial steady state liquid levels, the uniform system flow rate would have to be:

$$q_{vs} = q_{1s} = \frac{h_{1s} - h_{2s}}{R_1} = \frac{9-3}{2} = 3\ CFM$$

$$q_{vs} = q_{2s} = \frac{h_{2s}}{R_2} = \frac{3}{1} = 3\ CFM$$

With the system initially at steady state conditions, we will proceed to study the control scenario for maintaining the set point liquid level h_2 in the second tank at 3 feet when suddenly a second feed flow rate of 2.5 CFM is added to the first tank.

The BASIC program listing (CNTR2.BAS), which simulates the 2-interacting tank control scenario, is provided in Table 5. The format of this program is quite similar to that of BASIC program CNTR1A.BAS written for the heated tank control system. Lines 20-40 instruct the program to input system parameters, initial liquid levels, initial uniform inlet/outlet flow rates and the additional flow disturbance rate to the first tank. Line 50 calls for the integration interval, time duration and printout frequency parameter. For the specific run to be executed here:

H = 0.1 min ; M = 20 min ; S = 40

Then from Line 180 we would compute:

$$\frac{M}{(H)(S)} = \frac{20}{(0.1)(40)} = 5$$

Therefore, Line 370 prints the results line by line every (0.1)(5) or 0.5 min up to a total time duration of 20 minutes.

The initial conditions are set in Lines 120-125. The DEF function statements for the ODE's to be solved here (Eqns. 31, 32 and error integral) are declared in Lines 150-170. The repetitive RK integrations are executed in Lines 190-330. Line 340 determines the corrected inlet flow rate via Equation 33. The error, in this case, is h_{sp} - h_2, the deviation of the level in tank no. 2 from the set point.

We found that the following set of PI control parameters produced a satisfactory and stable solution:

$$K_c = 1.0 \text{ CFM/ft}$$

$$\tau_I = 1.5 \text{ min}$$

Here the effect of any transducer(s) are absorbed in the specified units for the proportional gain i.e. a direct conversion from height of liquid (ft) to feed flow rate (CFM).

Table 6 (CNTR2.OUT) lists the detailed tabular output. Figures 9 and 10 provide the respective liquid level and major feed stream flow rate profiles as a function of time. The liquid levels in each tank exhibit mild oscillatory behavior, and restore to their initial values quite nicely after about 20 minutes. The main feed flow rate lines out at a value very close to 0.5 CFM in order to counteract the effect of the inlet flow disturbance of 2.5 CFM. Thus the original net flow rate of 3 CFM throughout the entire system is restored once again.

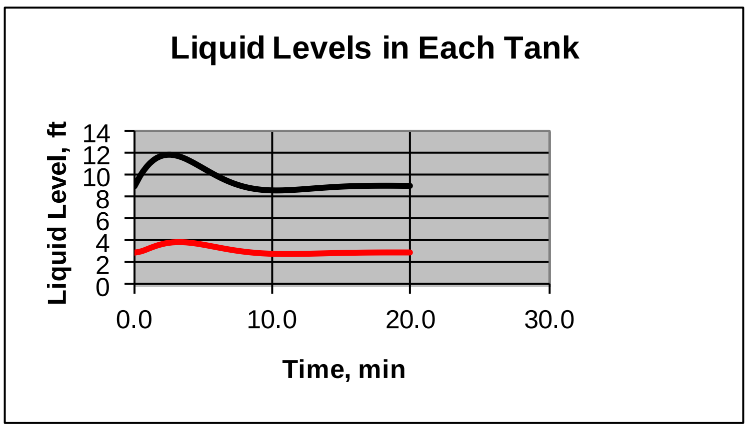

Figure 9 Illus. no. 2 (PI control action)

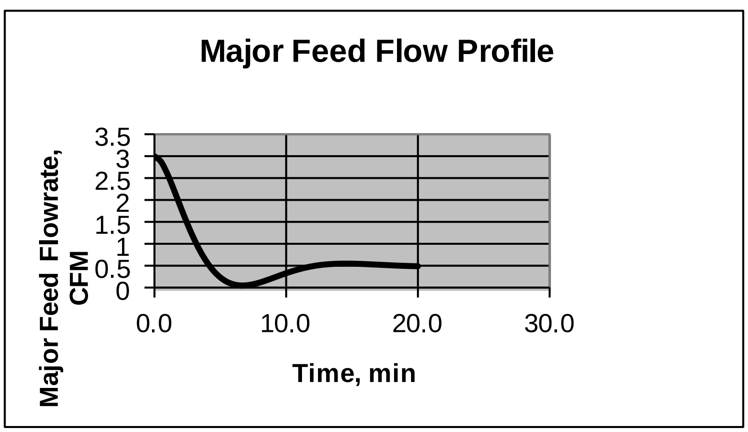

Figure 10 Illus. no. 2 (PI control action)

Illustration 3 The final illustration here was patterned after a two- CSTR chemical reactor control system studied by Coughanowr and Koppel (2) in their textbook "Process Systems Analysis and Control". This two-reactor in series network along with its proposed control system is shown in the block flow diagram below:

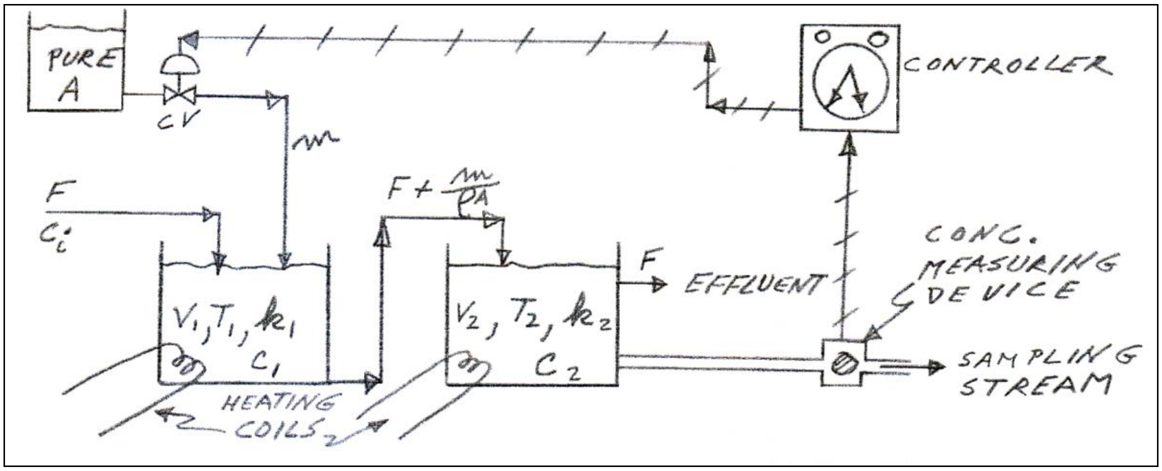

Diagram 6 Two-reactor in series network along with control system

A liquid feed stream is introduced to reactor no. 1 at a rate of F CFM. It contains a reactant A at a concentration of c_i lbmoles of A/ cuft. In both reactors, the reactant A is decomposing according to the irreversible elementary first-order reaction:

$$A \rightarrow B$$

The reaction is first-order with respect to the reactant decomposition and proceeds at a rate of,

$$r = -\frac{dc}{dt} = kc \qquad (34)$$

All of the pertinent parameters and variables involved in this process control model are defined below:

r = rate of reaction of species A reacting irreversibly in either reactor, lbmoles/cuft-min

k_1 = reaction rate constant for reactor no. 1, min^{-1}, varying with temperature.

k_2 = reaction rate constant for reactor no. 2, min^{-1}, varying with temperature.

c_i = concentration of reactant A in the feed to the first reactor, lbmoles/cuft.

c_1 = uniform concentration of species A in reactor no. 1 and exit stream at any time, lbmoles/cuft.

c_2 = uniform concentration of species A in reactor no. 2 and exit stream at any time, lbmoles/cuft.

m = flow rate of pure reactant A to reactor no. 1 (variable) for the purpose of process control, lbmoles/min.

V_1 = volume of reacting mixture (a constant holdup) in reactor no. 1, cuft.

V_2 = volume of reacting mixture (a constant holdup) in reactor no. 2, cuft.

F = Constant feed volumetric flow rate to reactor no. 1 and essentially constant throughout the entire reactor network, cuft/min.

The reaction is to be conducted in a series of two stirred (CSTR) reactors. The reactors are maintained at different temperatures such that the temperature in reactor no. 2 is to be greater than the temperature in reactor no. 1. As a result, k_2 will be greater than k_1. Any changes in physical properties of the fluids due to chemical reaction will be neglected.

The purpose of the control system here is to maintain c_2, the concentration of A leaving reactor no. 2 at some desired value (set point) regardless of any upset or perturbation in inlet concentration c_i. This control feature will be accomplished by adding a stream of pure reactant A (m, lbmoles/min) to reactor no. 1 through a control valve.

As in the previous illustrations, we first need to develop the dynamic mathematical model for the system which shows how the concentrations of reactant A from each reactor vary as a function of time. This basically consists of writing material balances with respect to reactant A around each reactor.

For reactor no. 1 using the notation defined above:

$$F\,c_i - \left(F + \frac{m}{\rho_A}\right)c_1 - k_1 V_1 c_1 + m = V_1 \frac{d\,c_1}{d\,t}$$

where m = molar flow rate of pure A through the valve, lbmoles/min

ρ_A = density of pure A, lbmoles/cuft

V_1 = volume holdup in reactor no. 1

Realistically, it can justifiably be assumed that volumetric flow rate of A through the control valve is much less than the inlet flow rate F i.e. $m/\rho_A << F$. As a result, the above material balance can be simplified to give,

$$F\,c_i - \left(F + k_1 V_1\right)c_1 + m = V_1 \frac{d\,c_1}{d\,t}$$

$$\textit{Next define:}\quad K_1 = \frac{F}{F + k_1 V_1} \quad ; \quad \tau_1 = \frac{V_1}{F + k_1 V_1}$$

Therefore, the material balance above can be rearranged and simplified to give,

$$F\,c_i - (F + k_1 V_1)\,c_1 + m = V_1 \frac{d\,c_1}{d\,t}$$

or $$\frac{F}{F + k_1 V_1} c_i - c_1 + \frac{m}{F + k_1 V_1} = \frac{V_1}{F + k_1 V_1} \frac{d\,c_1}{d\,t}$$

The definitions above are now substituted into the above ODE to yield the final material balance ODE for the first reactor:

$$K_1\,c_i - c_1 + \frac{K_1}{F} m = \tau_1 \frac{d\,c_1}{d\,t}$$

or $$\frac{d\,c_1}{d\,t} = \frac{1}{\tau_1}\left(K_1\,c_i - c_1 + \frac{K_1}{F} m\right) \qquad (35)$$

A material balance around reactor no. 2 with respect to reactant A yields,

$$F\,c_1 - F\,c_2 - k_2\,V_2\,c_2 = V_2 \frac{d\,c_2}{d\,t}$$

or $$\frac{F}{F + k_2 V_2} c_1 - c_2 = \frac{V_2}{F + k_2 V_2} \frac{d\,c_2}{d\,t}$$

and therefore $$\frac{d\,c_2}{d\,t} = \frac{1}{\tau_2}\left(K_2\,c_1 - c_2\right) \qquad (36)$$

where $$K_2 = \frac{F}{F + k_2 V_2} \quad ; \quad \tau_2 = \frac{V_2}{F + k_2 V_2}$$

As in the other illustrations, we will apply PI control for this network of reactors. The controller action for adjusting the supplemental feed flow rate of pure A to the first reactor is represented by the expression,

$$m = m_s + K_c\,\varepsilon + \frac{K_c}{\tau_I}\int_0^t \varepsilon\,dt \qquad (37)$$

where

$$\varepsilon = c_{2sp} - c_2\,,\ lbmoles/cuft$$

$$K_c = proportional\ \ gain,\ cuft/\min(CFM)$$

$$\tau_I = \text{int}\,egral\ \ time, \min$$

$$m_s = steady\ \ state\ \ flowrate\ of\ \ pure\ \ A$$
$$to\ \ first\ \ reactor,\ lbmoles/\min$$

Here are the numerical parameters and initial operating conditions for the reactor network:

c_i = 0.1 lb moles of A /cu ft

F = 100 CFM

m_s = 1.0 lb mole/min

k_1 = 1/6 min^{-1}

k_2 = 2/3 min^{-1}

$V_1 = V_2$ = 300 cu ft

At this point the constant parameters for Equations 35 and 36 can readily be computed.

$$K_1 = \frac{100}{100 + \frac{1}{6}(300)} = \frac{2}{3}\ \ ;\ \ \tau_1 = \frac{300}{100 + \frac{1}{6}(300)} = 2.0\ \min$$

$$K_2 = \frac{100}{100 + \frac{2}{3}(300)} = \frac{1}{3}\ \ ;\ \ \tau_2 = \frac{300}{100 + \frac{2}{3}(300)} = 1.0\ \min$$

These parameters will be calculated internally in the BASIC program to be described shortly.

The initial reactant concentrations in the two reactors are computed by setting

$$\frac{d\,c_1}{d\,t} = \frac{d\,c_2}{d\,t} = 0$$

in Equations 35 and 36 and then solving them explicitly for c_{1s} and c_{2s}

.

$$c_{1s} = K_1\,c_{is} + \frac{K_1}{F}\,m_s \qquad (38)$$

and

$$c_{2s} = K_2\,c_{1s} \qquad (39)$$

and thus

$$c_{1s} = \frac{2}{3}(0.1) + \frac{2/3}{100}(1.0) = 0.0733\ lb\,moles\ A/cu\ ft$$

$$c_{2s} = \frac{1}{3}(0.0733) = 0.0244\ lb\,moles\ A/cu\ ft$$

Now we will study the control scenario for this system when the inlet concentration of reactant A in the main feed stream to reactor no. 1 is reduced from 0.1 to 0.05 lb moles A/ cu ft.

Table 7 lists the BASIC program (CNTR3.BAS) which simulates the 2-reactor control scenario. Once again, the general format here is basically the same as for the programs written for the previous two illustrations. Line 10 reads in the values for system parameters and initial conditions. Line 15 calls for the integration interval, time duration and printout frequency parameter. For the specific run to be executed here we have:

H = 0.1 min ; M0 = 100 min ; S = 100

an from Line 160 the program will then compute:

$$\frac{M0}{(H)(S)} = \frac{100}{(0.1)(100)} = 10$$

As a result, Line 255 will print the results line by line every (0.1)(10) or 1.0 min up to a total time duration of 100 minutes.

The initial reactant concentrations from each reactor are computed via Lines 65-90. Trial values for the proportional gain and integral time are inputted via Lines 95-110. The DEF function statements for the ODE's to be solved here (Eqns. 35, 36 and error integral) are declared in Lines 150-157. The repetitive RK integrations are executed in Lines 165-235. In Line 240 the corrected flow rate of the pure reactant A stream to reactor no. 1 is computed via Equation 37. The error in this case is c_{2s} - c_2, the deviation of the variable concentration of reactant A in reactor no. 2 effluent from the specified set point.

A satisfactory and stable control solution was obtained by selecting the following PI control parameters:

$$Kc = 50 \text{ CFM}$$

$$\tau_I = 2 \text{ min}$$

Table 8 (CNTR3.OUT) lists the entire output. Figures 11 and 12 show the concentration profiles for each reactor and history of the supplemental feed flow rate of pure reactant A as a function of time. After about 5 minutes, both reactor effluent concentrations seek a minimum value and then rise steadily back to their initial desired values (set point). The system appears to stabilize nicely after about 50 minutes of elapsed time. The supplemental feed flow rate levels out at a final value which is very close to 6.0 lb moles/min.

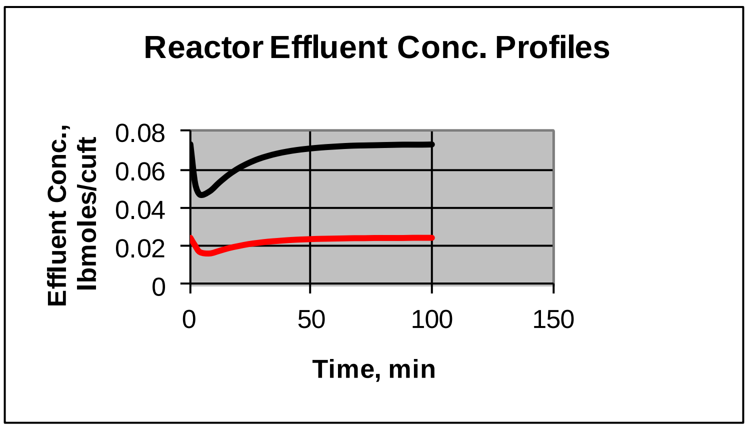

Figure 11 Illus. no. 3 (PI control action)

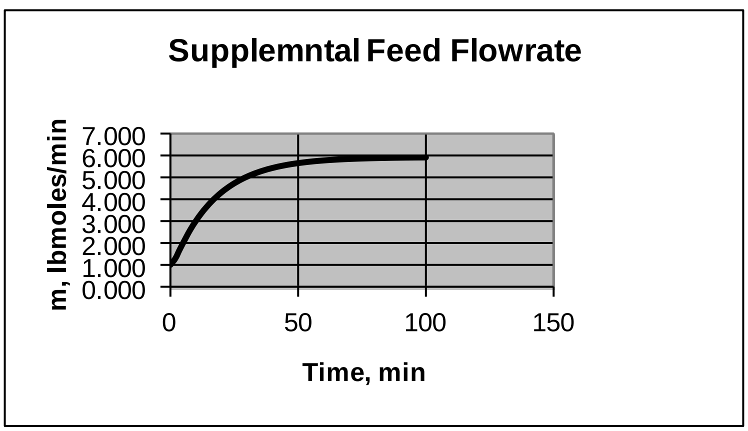

Figure 12 Illus. no. 3 (PI control action)

And finally, in Figure 13 we have plotted the reactor effluent concentration profiles for the case where no control action is applied (open loop operation). In about 10 minutes the reactant concentration in reactor no. 1 is reduced from 0.0733 lb moles/cu ft to a final steady state value of 0.0401 lb moles/ cu ft. In reactor no. 2 the reactant concentration drops from 0.0244 to 0.0134 lb moles/ cu ft.

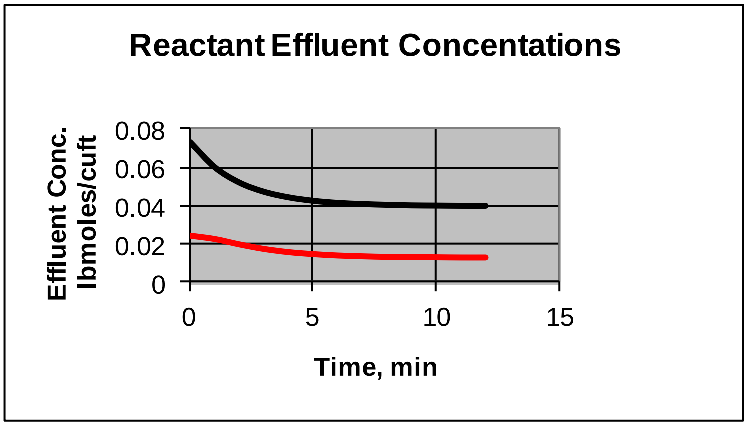

Figure 13 Illus. no. 3 (no control action - open loop)

References

1. Koppany, C.R., "A Useful BASIC Algorithm for Solving ODE's Subject to a Specified Set of Initial Conditions", (Aug. 8, 2012) (See the Website CRKTECH.COM.

2. Coughanowr, D.R. and Koppel, L.B. "Process Systems Analysis and Control", McGraw-Hill Book Co., (1965).

3. Harriott, P., "Process Control", McGraw-Hill Book Co. (1964).

4. Shinskey, F.G. "Process-Control Systems", McGraw-Hill Book Co., (1967).

Appendix

Table 1
BASIC Program Listing for Illus. No. 1 (Case A)

```
5 OPEN "CNTR1A.OUT" FOR OUTPUT AS #1
10 REM "CLOSED LOOP CONTROL-HEATED TANK"
15 REM "PERTURB INLET STREAM TEMP."
17 REM "CONTROL DESIRED OUTLET TEMP.(SET PT.)"
20 READ TIS, TI, TR, W, CP, D, V
30 READ H, M, S
40 QS = W * CP * (TR - TIS)
50 PRINT #1, "STEADY STATE Q, BTU/SEC="; QS
60 PRINT #1, ""
70 PRINT #1, "INITIAL STEADY STATE TIS="; TIS
72 PRINT #1, ""
75 PRINT #1, "NEW INLET TEMP, TI="; TI
80 PRINT #1, ""
90 INPUT "PROPORTIONAL GAIN KC ="; KC
100 PRINT #1, "PROPORTIONAL GAIN KC="; KC
110 PRINT #1, ""
120 INPUT "INTEGRAL TIME IT="; IT
130 PRINT #1, "INTEGRAL TIME IT ="; IT
140 PRINT #1, ""
150 PRINT #1, "TIME,SEC"; TAB(13); "TEMP,F"; TAB(26); " HEAT, BTU / SEC"
160 TIME = 0
165 T = TR
167 Q = QS
170 PRINT #1, USING "#####.##"; TIME; TAB(13); T; TAB(26); Q
175 Z = 0
180 C1 = W / (D * V)
190 C2 = D * V * CP
210 DEF FNA (T) = C1 * (TI - T) + Q / C2
229 DEF FNB (T) = TR - T
230 FOR T9 = 1 TO M / (S * H)
240 K1 = H * FNA(T)
250 L1 = H * FNB(T)
260 K2 = H * FNA(T + K1 / 2)
270 L2 = H * FNB(T + K1 / 2)
280 K3 = H * FNA(T + K2 / 2)
290 L3 = H * FNB(T + K2 / 2)
300 K4 = H * FNA(T + K3)
310 L4 = H * FNB(T + K3)
320 T = T + (K1 + 2 * K2 + 2 * K3 + K4) / 6
330 Z = Z + (L1 + 2 * L2 + 2 * L3 + L4) / 6
340 Q = QS + KC * ((TR - T) + Z / IT)
350 TIME = TIME + H
360 NEXT T9
370 PRINT #1, USING "#####.##"; TIME; TAB(13); T; TAB(26); Q
380 IF TIME > M THEN 999
390 GOTO 230
500 DATA 100,150,300,100,1,60,100
510 DATA 0.1,400,200
999 END
```

Table 2
Output (CNTR1A.OUT) for Illus.1-Case A

STEADY STATE Q, BTU/SEC= 20000
INITIAL STEADY STATE TIS= 100
NEW INLET TEMP, TI= 150
PROPORTIONAL GAIN KC= 7
INTEGRAL TIME IT = .2

TIME,SEC	TEMP,F	HEAT, BTU / SEC
0.00	300.00	20000.00
2.00	301.63	19931.03
4.00	303.17	19751.59
6.00	304.58	19469.60
8.00	305.84	19095.05
10.00	306.92	18639.69
12.00	307.81	18116.74
14.00	308.49	17540.49
16.00	308.94	16925.96
18.00	309.18	16288.54
20.00	309.20	15643.61
22.00	309.01	15006.18
24.00	308.62	14390.59
26.00	308.05	13810.17
28.00	307.31	13276.98
30.00	306.43	12801.61
32.00	305.43	12392.94
34.00	304.34	12058.02
36.00	303.19	11801.97
38.00	302.01	11627.94
40.00	300.82	11537.09
42.00	299.66	11528.61
44.00	298.54	11599.86
46.00	297.49	11746.48
48.00	296.54	11962.50
50.00	295.70	12240.58
52.00	294.99	12572.23
54.00	294.41	12948.00
56.00	293.99	13357.78
58.00	293.71	13791.01
60.00	293.59	14236.98
62.00	293.62	14685.03
64.00	293.80	15124.84
66.00	294.11	15546.63
68.00	294.54	15941.37
70.00	295.09	16300.96
72.00	295.73	16618.37
74.00	296.44	16887.81
76.00	297.21	17104.76
78.00	298.02	17266.05
80.00	298.85	17369.92
82.00	299.67	17415.98
84.00	300.47	17405.16
86.00	301.24	17339.67
88.00	301.95	17222.89
90.00	302.59	17059.26
92.00	303.15	16854.15
94.00	303.61	16613.68
96.00	303.98	16344.57
98.00	304.25	16053.96
100.00	304.41	15749.25
102.00	304.46	15437.91

Table 3
BASIC Program Listing for Illus. No. 1 (Case B)

```
5 OPEN "CNTR1B.OUT" FOR OUTPUT AS #1
10 REM "CLOSED LOOP CONTROL-HEATED TANK"
15 REM "PERTURB DISCHARGE STREAM TEMP.TO NEW VALUE"
17 REM "CONTROL DESIRED OUTLET TEMP.(SET PT.)"
20 READ TIS, TRS, TR, W, CP, D, V
30 READ H, M, S
40 QS = W * CP * (TRS - TIS)
50 PRINT #1, "STEADY STATE Q, BTU/SEC="; QS
60 PRINT #1, ""
70 PRINT #1, "INITIAL DISCHARGE STREAM TEMP,F="; TRS
72 PRINT #1, ""
75 PRINT #1, "NEW DISCHARGE STREAM TEMP,F(SET PT.)="; TR
80 PRINT #1, ""
90 INPUT "PROPORTIONAL GAIN KC ="; KC
100 PRINT #1, "PROPORTIONAL GAIN KC="; KC
110 PRINT #1, ""
120 INPUT "INTEGRAL TIME IT="; IT
130 PRINT #1, "INTEGRAL TIME IT ="; IT
140 PRINT #1, ""
150 PRINT #1, "TIME,SEC"; TAB(13); "TEMP,F"; TAB(26); " HEAT, BTU / SEC"
160 TIME = 0
165 T = TRS
167 Q = QS
170 PRINT #1, USING "#####.##"; TIME; TAB(13); T; TAB(26); Q
175 Z = 0
180 C1 = W / (D * V)
190 C2 = D * V * CP
210 DEF FNA (T) = C1 * (TIS - T) + Q / C2
229 DEF FNB (T) = TR - T
230 FOR T9 = 1 TO M / (S * H)
240 K1 = H * FNA(T)
250 L1 = H * FNB(T)
260 K2 = H * FNA(T + K1 / 2)
270 L2 = H * FNB(T + K1 / 2)
280 K3 = H * FNA(T + K2 / 2)
290 L3 = H * FNB(T + K2 / 2)
300 K4 = H * FNA(T + K3)
310 L4 = H * FNB(T + K3)
320 T = T + (K1 + 2 * K2 + 2 * K3 + K4) / 6
330 Z = Z + (L1 + 2 * L2 + 2 * L3 + L4) / 6
340 Q = QS + KC * ((TR - T) + Z / IT)
350 TIME = TIME + H
360 NEXT T9
370 PRINT #1, USING "#####.##"; TIME; TAB(13); T; TAB(26); Q
380 IF TIME > M THEN 999
390 GOTO 230
500 DATA 100,300,350,100,1,60,100
510 DATA 0.1,400,200
999 END
```

Table 4
Output (CNTR1B.OUT) for Illus.1-Case B

STEADY STATE Q, BTU/SEC= 20000
INITIAL DISCHARGE STREAM TEMP,F= 300
NEW DISCHARGE STREAM TEMP,F(SET PT.)= 350
PROPORTIONAL GAIN KC= 7
INTEGRAL TIME IT = .2

TIME,SEC	TEMP,F	HEAT, BTU / SEC
0.00	300.00	20000.00
4.00	302.42	27215.38
8.00	308.94	33417.36
12.00	318.69	38446.20
16.00	330.58	41931.63
20.00	343.40	43666.50
24.00	355.95	43614.30
28.00	367.10	41900.79
32.00	375.98	38791.79
36.00	381.93	34659.95
40.00	384.62	29944.10
44.00	384.03	25105.62
48.00	380.44	20585.57
52.00	374.37	16766.65
56.00	366.53	13942.90
60.00	357.74	12299.43
64.00	348.86	11903.26
68.00	340.68	12705.30
72.00	333.89	14552.62
76.00	329.00	17209.16
80.00	326.32	20382.49
84.00	325.94	23753.76
88.00	327.74	27008.04
92.00	331.40	29862.19
96.00	336.48	32088.04
100.00	342.43	33529.11
104.00	348.65	34109.67
108.00	354.57	33836.13
112.00	359.68	32790.82
116.00	363.58	31119.53
120.00	366.00	29014.18
124.00	366.82	26692.46
128.00	366.09	24376.69
132.00	363.96	22273.59
136.00	360.73	20556.87
140.00	356.76	19353.88
144.00	352.46	18737.39
148.00	348.22	18722.79
152.00	344.43	19270.62
156.00	341.40	20293.74
160.00	339.34	21668.46
164.00	338.38	23247.99
168.00	338.52	24877.19
172.00	339.67	26406.96
176.00	341.67	27707.23
180.00	344.29	28677.25
184.00	347.23	29252.65
188.00	350.22	29408.70
192.00	353.00	29159.88
196.00	355.31	28555.84
200.00	356.99	27674.71
204.00	357.94	26614.06
208.00	358.11	25480.92
212.00	357.54	24381.54
216.00	356.34	23411.99

220.00	354.65	22650.16
224.00	352.66	22150.11
228.00	350.57	21938.84
232.00	348.57	22015.68
236.01	346.83	22354.23
240.01	345.49	22906.47
244.01	344.65	23608.54
248.01	344.34	24387.49
252.01	344.56	25168.44
256.01	345.25	25881.40
260.01	346.32	26467.23
264.01	347.65	26882.12
268.01	349.09	27100.57
272.01	350.52	27116.34
276.01	351.81	26941.76
280.01	352.84	26605.35
284.01	353.56	26148.15
288.01	353.90	25619.24
292.01	353.88	25070.79
296.01	353.50	24553.18
300.01	352.84	24110.60
304.01	351.97	23777.55
308.01	350.99	23576.40
312.01	349.98	23516.19
316.01	349.04	23592.77
320.01	348.25	23790.03
324.01	347.67	24082.14
328.01	347.34	24436.54
332.01	347.27	24817.32
336.01	347.45	25188.63
340.01	347.84	25517.91
344.01	348.40	25778.56
348.01	349.07	25951.92
352.01	349.77	26028.38
356.01	350.45	26007.69
360.01	351.04	25898.29
364.01	351.50	25715.95
368.01	351.79	25481.93
372.01	351.90	25220.67
376.01	351.84	24957.38
380.01	351.61	24715.74
384.01	351.26	24515.90
388.01	350.82	24372.91
392.01	350.33	24295.70
396.01	349.85	24286.71
400.02	349.41	24342.13

Table 5
BASIC Program Listing for Illus. No. 2

```
5 OPEN "CNTR2.OUT" FOR OUTPUT AS #1
10 REM "CLOSED LOOP CONTROL - 2 INTERACTING TANKS"
20 READ A1, A2, R1, R2
30 READ H10, H20
40 READ QS, L
50 READ H, M, S
60 PRINT #1, "STEADY STATE FLOWRATE="; QS
70 PRINT #1, ""
75 PRINT #1, "LOAD DISTURBANCE FLOWRATE="; L
80 PRINT #1, ""
85 INPUT "PROPORTIONAL GAIN KC="; KC
90 PRINT #1, "PROPORTIONAL GAIN KC="; KC
95 PRINT #1, ""
100 INPUT "INTEGRAL TIME IT="; IT
105 PRINT #1, "INTEGRAL TIME IT="; IT
110 PRINT #1, ""
115 PRINT #1, "TIME"; TAB(13); "H1"; TAB(26); "H2"; TAB(39); "Q"
120 H1 = H10
122 H2 = H20
123 Z = 0
125 T = 0
127 Q = QS + L
130 PRINT #1, USING "##.###"; T; TAB(13); H1; TAB(26); H2; TAB(39); QS
150 DEF FNA (H1, H2) = (Q / A1) - (H1 - H2) / (R1 * A1)
160 DEF FNB (H1, H2) = (H1 - H2) / (R1 * A2) - H2 / (R2 * A2)
170 DEF FNC (H2) = H20 - H2
180 FOR T1 = 1 TO M / (S * H)
190 K1 = H * FNA(H1, H2)
200 L1 = H * FNB(H1, H2)
210 M1 = H * FNC(H2)
220 K2 = H * FNA(H1 + K1 / 2, H2 + L1 / 2)
230 L2 = H * FNB(H1 + K1 / 2, H2 + L1 / 2)
240 M2 = H * FNC(H2 + L1 / 2)
250 K3 = H * FNA(H1 + K2 / 2, H2 + L2 / 2)
260 L3 = H * FNB(H1 + K2 / 2, H2 + L2 / 2)
270 M3 = H * FNC(H2 + L2 / 2)
280 K4 = H * FNA(H1 + K3, H2 + L3)
290 L4 = H * FNB(H1 + K3, H2 + L3)
300 M4 = H * FNC(H2 + L3)
310 H1 = H1 + (K1 + 2 * K2 + 2 * K3 + K4) / 6
320 H2 = H2 + (L1 + 2 * L2 + 2 * L3 + L4) / 6
330 Z = Z + (M1 + 2 * M2 + 2 * M3 + M4) / 6
340 QV = QS + KC * (H20 - H2 + Z / IT)
345 Q = QV + L
350 T = T + H
360 NEXT T1
370 PRINT #1, USING "##.###"; T; TAB(13); H1; TAB(26); H2; TAB(39); QV
380 IF T > M THEN 999
390 GOTO 180
400 DATA 1,1,2,1
410 DATA 9,3
420 DATA 3,2.5
430 DATA 0.1,20,40
999 END
```

Table 6
Output (CNTR2OUT) For Illus. No. 2

STEADY STATE FLOWRATE= 3

LOAD DISTURBANCE FLOWRATE= 2.5

PROPORTIONAL GAIN KC= 1

INTEGRAL TIME IT= 1.5

TIME	H1	H2	Q
0.000	9.000	3.000	3.000
0.500	10.100	3.113	2.873
1.000	10.906	3.331	2.583
1.500	11.439	3.548	2.219
2.000	11.730	3.720	1.834
2.500	11.819	3.831	1.462
3.000	11.748	3.883	1.122
3.500	11.555	3.883	0.827
4.000	11.278	3.840	0.582
4.500	10.950	3.766	0.388
5.000	10.600	3.670	0.244
5.500	10.249	3.563	0.145
6.000	9.915	3.452	0.087
6.500	9.612	3.344	0.062
7.000	9.347	3.243	0.066
7.500	9.125	3.153	0.090
8.000	8.947	3.076	0.129
8.500	8.813	3.013	0.178
9.000	8.719	2.963	0.232
9.500	8.660	2.927	0.287
10.000	8.632	2.902	0.340
10.500	8.628	2.889	0.389
11.000	8.644	2.883	0.432
11.500	8.674	2.885	0.469
12.000	8.714	2.892	0.499
12.500	8.759	2.903	0.523
13.000	8.806	2.917	0.539
13.500	8.852	2.931	0.550
14.000	8.894	2.946	0.556
14.500	8.933	2.960	0.558
15.000	8.966	2.972	0.557
15.500	8.993	2.984	0.553
16.000	9.014	2.993	0.547
16.500	9.030	3.001	0.540
17.000	9.041	3.007	0.533
17.500	9.047	3.011	0.526
18.000	9.049	3.014	0.519
18.500	9.049	3.015	0.512
19.000	9.046	3.015	0.507
19.500	9.041	3.015	0.502
20.000	9.036	3.014	0.499

Table 7
BASIC Program Listing for Illus. No. 3

```
5 OPEN "CNTR3.OUT" FOR OUTPUT AS #1
10 READ CIS, CI, V1, V2, F, K1, K2, MS
12 REM "CLOSED LOOP CONTROL - 2 CSTRS IN SERIES"
15 READ H, M0, S
20 PRINT #1, "INITIAL CIN, MOLS/CUFT="; CIS
25 PRINT #1, ""
30 PRINT #1, "NEW CIN, MOLS/CUFT ="; CI
35 PRINT #1, " CSTR NO. 1 TANK VOL., CUFT = "; V1
40 PRINT #1, " CSTR nO. 2 TANK VOL., CUFT = "; V2
45 PRINT #1, "VOL. FEED RATE, CFM="; F
50 PRINT #1, "REACTOR NO. 1 RATE CONST. 1/MIN="; K1
55 PRINT #1, "REACTOR NO. 2 RATE CONST. 1/MIN="; K2
60 PRINT #1, "INITIAL SUPPLE. STREAM FLOWRATE,MOLS/HR="; MS
65 K11 = F / (F + K1 * V1)
70 TAU1 = V1 / (F + K1 * V1)
75 K22 = F / (F + K2 * V2)
80 TAU2 = V2 / (F + K2 * V2)
85 C1S = K11 * CIS + (K11 / F) * MS
90 C2S = K22 * C1S
95 INPUT "PROPORTIONAL GAIN KC ="; KC
100 PRINT #1, "PROPORTIONAL GAIN KC="; KC
105 PRINT #1, ""
110 INPUT "INTEGRAL TIME IT="; IT
115 PRINT #1, "INTEGRAL TIME IT ="; IT
120 PRINT #1, ""
125 PRINT #1, "TIME,MIN"; TAB(13); "C1,M/CUFT"; TAB(26); "C2,M/CUFT"; TAB(39);
"M,M/MIN"
130 PRINT #1, ""
135 TIME = 0
140 PRINT #1, USING "##.####"; TIME; TAB(13); C1S; TAB(26); C2S; TAB(39); MS
142 C1 = C1S
144 C2 = C2S
145 Z = 0
152 M = MS
150 DEF FNA (C1) = (K11 * CI - C1 + (K11 / F) * M) / TAU1
155 DEF FNB (C1, C2) = (K22 * C1 - C2) / TAU2
157 DEF FNC (C2) = C2S - C2
160 FOR T9 = 1 TO M0 / (S * H)
165 L1 = H * FNA(C1)
170 M1 = H * FNB(C1, C2)
175 N1 = H * FNC(C2)
180 L2 = H * FNA(C1 + L1 / 2)
185 M2 = H * FNB(C1 + L1 / 2, C2 + M1 / 2)
190 N2 = H * FNC(C2 + M1 / 2)
195 L3 = H * FNA(C1 + L2 / 2)
200 M3 = H * FNB(C1 + L2 / 2, C2 + M2 / 2)
205 N3 = H * FNC(C2 + M2 / 2)
210 L4 = H * FNA(C1 + L3)
215 M4 = H * FNB(C1 + L3, C2 + M3)
220 N4 = H * FNC(C2 + M3)
225 C1 = C1 + (L1 + 2 * L2 + 2 * L3 + L4) / 6
230 C2 = C2 + (M1 + 2 * M2 + 2 * M3 + M4) / 6
235 Z = Z + (N1 + 2 * N2 + 2 * N3 + N4) / 6
240 M = MS + KC * (C2S - C2 + Z / IT)
245 TIME = TIME + H
250 NEXT T9
255 PRINT #1, USING "##.####"; TIME; TAB(13); C1; TAB(26); C2; TAB(39); M
260 IF TIME > M0 THEN 999
265 GOTO 160
300 DATA 0.1,0.05,300,300,100,0.167,0.667,1
310 DATA 0.1,100,100
999 END
```

Table 8
Output (CNTR3.OUT) For Illus. No. 3

```
INITIAL CIN, MOLS/CUFT= .1

NEW CIN, MOLS/CUFT = .05
 CSTR NO. 1 TANK VOL., CUFT =  300
 CSTR nO. 2 TANK VOL., CUFT =  300
VOL. FEED RATE, CFM= 100
REACTOR NO. 1 RATE CONST. 1/MIN= .167
REACTOR NO. 2 RATE CONST. 1/MIN= .667
INITIAL SUPPLE. STREAM FLOWRATE,MOLS/HR= 1
PROPORTIONAL GAIN KC= 50

INTEGRAL TIME IT = 2
```

TIME,MIN	C1,M/CUFT	C2,M/CUFT	M,M/MIN
0.0000	0.0733	0.0244	1.0000
1.0000	0.0603	0.0227	1.1018
2.0000	0.0528	0.0201	1.3107
3.0000	0.0489	0.0180	1.5518
4.0000	0.0471	0.0167	1.7962
5.0000	0.0467	0.0160	2.0334
6.0000	0.0471	0.0158	2.2596
7.0000	0.0479	0.0158	2.4738
8.0000	0.0489	0.0160	2.6761
9.0000	0.0500	0.0163	2.8669
10.0000	0.0512	0.0167	3.0469
11.0000	0.0524	0.0171	3.2165
12.0000	0.0535	0.0174	3.3764
13.0000	0.0546	0.0178	3.5271
14.0000	0.0557	0.0182	3.6691
15.0000	0.0567	0.0185	3.8030
16.0000	0.0576	0.0189	3.9292
17.0000	0.0585	0.0192	4.0482
18.0000	0.0593	0.0195	4.1603
19.0000	0.0601	0.0198	4.2660
20.0000	0.0609	0.0200	4.3656
21.0000	0.0616	0.0203	4.4595
22.0000	0.0623	0.0205	4.5480
23.0001	0.0629	0.0207	4.6314
24.0001	0.0635	0.0210	4.7100
25.0001	0.0641	0.0212	4.7841
26.0001	0.0646	0.0213	4.8539
27.0001	0.0651	0.0215	4.9198
28.0001	0.0656	0.0217	4.9818
29.0001	0.0660	0.0218	5.0403
30.0001	0.0664	0.0220	5.0954
31.0001	0.0668	0.0221	5.1474
32.0001	0.0672	0.0223	5.1964
33.0001	0.0675	0.0224	5.2425
34.0001	0.0679	0.0225	5.2860
35.0000	0.0682	0.0226	5.3270
36.0000	0.0685	0.0227	5.3657
37.0000	0.0688	0.0228	5.4021
38.0000	0.0690	0.0229	5.4365
39.0000	0.0693	0.0230	5.4689
40.0000	0.0695	0.0231	5.4994
40.9999	0.0697	0.0232	5.5281
41.9999	0.0699	0.0232	5.5552
42.9999	0.0701	0.0233	5.5808
43.9999	0.0703	0.0234	5.6049
44.9999	0.0705	0.0234	5.6276

45.9999	0.0706	0.0235	5.6489
46.9999	0.0708	0.0235	5.6691
47.9998	0.0709	0.0236	5.6881
48.9998	0.0711	0.0236	5.7060
49.9998	0.0712	0.0237	5.7229
50.9998	0.0713	0.0237	5.7388
51.9998	0.0714	0.0238	5.7538
52.9998	0.0715	0.0238	5.7680
53.9997	0.0716	0.0238	5.7813
54.9997	0.0717	0.0239	5.7939
55.9997	0.0718	0.0239	5.8057
56.9997	0.0719	0.0239	5.8169
57.9997	0.0720	0.0240	5.8274
58.9997	0.0721	0.0240	5.8373
59.9997	0.0721	0.0240	5.8466
60.9996	0.0722	0.0240	5.8555
61.9996	0.0723	0.0241	5.8638
62.9996	0.0723	0.0241	5.8716
63.9996	0.0724	0.0241	5.8790
64.9996	0.0724	0.0241	5.8859
65.9996	0.0725	0.0241	5.8925
66.9995	0.0725	0.0241	5.8986
67.9995	0.0726	0.0242	5.9045
68.9995	0.0726	0.0242	5.9100
69.9995	0.0726	0.0242	5.9151
70.9995	0.0727	0.0242	5.9200
71.9995	0.0727	0.0242	5.9246
72.9995	0.0727	0.0242	5.9289
73.9994	0.0728	0.0242	5.9330
74.9994	0.0728	0.0243	5.9369
75.9994	0.0728	0.0243	5.9405
76.9994	0.0729	0.0243	5.9439
77.9994	0.0729	0.0243	5.9471
78.9994	0.0729	0.0243	5.9502
79.9994	0.0729	0.0243	5.9530
80.9993	0.0729	0.0243	5.9557
81.9993	0.0730	0.0243	5.9583
82.9993	0.0730	0.0243	5.9607
83.9993	0.0730	0.0243	5.9629
84.9993	0.0730	0.0243	5.9651
85.9993	0.0730	0.0243	5.9671
86.9992	0.0730	0.0243	5.9690
87.9992	0.0731	0.0243	5.9707
88.9992	0.0731	0.0243	5.9724
89.9992	0.0731	0.0244	5.9740
90.9992	0.0731	0.0244	5.9755
91.9992	0.0731	0.0244	5.9769
92.9992	0.0731	0.0244	5.9782
93.9991	0.0731	0.0244	5.9795
94.9991	0.0731	0.0244	5.9807
95.9991	0.0731	0.0244	5.9818
96.9991	0.0732	0.0244	5.9828
97.9991	0.0732	0.0244	5.9838
98.9991	0.0732	0.0244	5.9847
99.9990	0.0732	0.0244	5.9856
%100.9990	0.0732	0.0244	5.9864